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Quasi-Static Analysis of a Leg-Wheel Hybrid Vehicle for Enhancing Stair Climbing Ability

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***Abstract* - This paper presents quasi-static analysis of a leg-wheel hybrid vehicle for enhancing stair climbing ability. The vehicle consists of four spoke-wheels. The number of spokes for each wheel can be altered by varying the number of strips attaching to the wheel hub. The effects of having different number of spokes are studied in two bases: climbing ability over an obstacle and the required coefficient of friction between spoke tips and stairs. The climbing ability is measured by the maximum height of obstacle that the vehicle is able to travel. For a certain friction coefficient, the limit for obstacle height can also be predicted. Leg-wheel vehicles with four, six and twelve spokes are also built and tested. Results from the quasi-static analysis and the experiments are shown to be comparable. The slight difference is due to deformations of the spoke strips from the vehicle weight. Calculation and experimental results are in agreement. The maximum step height of the leg wheel can roll over in each case is relatively more than its hub. The twelve-spoke wheel gives the maximum absolute height among the three cases.**

Index Terms - Leg-Wheel hybrid vehicle, Stair climbing ability, Quasi-static analysis

I. INTRODUCTION

In recent years, a number of developments have been done on robots having different types of locomotion, depending on the environment. While wheel mechanisms are relatively simple and allow quick mobility, leg mechanisms have shown tremendous advantages in climbing over obstacles.

In early studies, for example, Raibert and his team successfully built monopod and biped robots that are able to hop over stairs. They also developed dynamic stability theory for use with legged vehicles [1]. TITAN III, Hirose and his team, were built to proof the theories of adaptive gain control in various type of terrains including stair-climbing ability. The robot consists of four legs with pantomec leg mechanism [2].

PROLERO (PROtotype of LEgged ROver) was developed in 1996. This robot used a total of 6 motors– one for each of its 6 legs. Each leg is comprised of an L-shaped spoke rotating in a full circle to move its body forward and lift it over an obstacle [3].

RHex is a compliant hexapod robot inspired by cockroach locomotion. This robot has the ability to traverse over rough terrain and climb stairs [4], [5], [6].

A recent robot, made by EPFL, is called Shrimp which has six motorized wheels with the ability to climb objects up to two times its wheel diameter [7].

Whegs^{TM} (Wheel+legs) robots followed the design of RHex as demonstrating the advantages of the combination of wheels and legs. Each wheg consists of a three-spoke configuration. The robot uses one motor for propulsion and is moved in an alternating tripod gait [8, 9].

Takahashi, et al. [9] also introduced a leg-wheel hybrid quadruped robot used load distribution to climb over obstacles by a twisting torque at the body center

This study presents a new leg-wheel quadruped hybrid vehicle similar to Whegs^{TM} but with adjustable number of spokes. The vehicle was built as shown in Fig. 1.

Fig. 1 A Leg-Wheel Hybrid Vehicle with adjustable number of spokes

Climbing ability of the vehicle with normal wheel and leg wheel with different number of spokes up to twelve is investigated. Method of climbing ability analysis is proposed by using quasi-static assumptions. The analysis involves evaluation of the maximum obstacle height that the vehicle can travel as well as the required coefficient of friction between the spoke tips and obstacles.

Experiments for the vehicle with four, six, and twelve spokes are performed in controlled field to obtain climbing ability in each case. Six-spoke wheel is chosen as a representative to be tested in a rescued terrain.

II. CLIMBING ANALYSIS

Climbing ability of a vehicle over an obstacle is examined by using quasi-static analysis based on two principles:

geometric consideration and static friction condition. The former offers the maximum obstacle height that the wheel is able to travel from geometric point of view, provided that the applied torque and the frictional force are sufficient. The latter proposes a method to approximate the step height which the vehicle can climb over for a specified applied torque and friction coefficient between the wheel and the obstacle.

1. *Geometric Consideration*

Schematic diagrams for step climbing of normal wheel and spoke wheel mechanisms are illustrated in Fig. 2(a) and 2(b), respectively. In this case, climbing ability depends mainly upon the wheel radius. It can be seen from Fig. 2(a) that unless the wheel climbs up vertically, the maximum obstacle height h is limited to the wheel radius r.

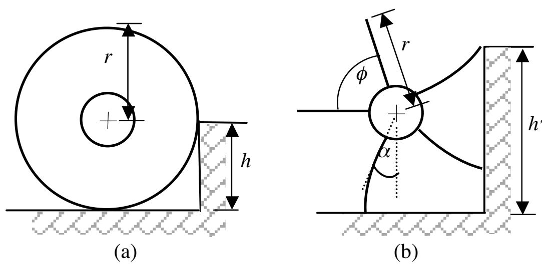

Fig. 2 Schematic diagram for (a) normal wheel (b) compliant spoke wheel

Fig. 2(b) shows the initial condition (position and orientation) to deliver the highest climbing ability for five-compliant-spoke wheel. The spokes are arranged evenly with an angle of ϕ and the wheel radius r is measured from the hub center to the tip of the spokes. The angle α is measured from the vertical line relative to the nearest spoke touching the ground where the clockwise direction gives positive angle. The maximum height of climbing for spoke wheel h' can then be expressed as

$$h' = r\left[\sin\left(\phi/2\right) + \cos\alpha\right] \tag{1}$$

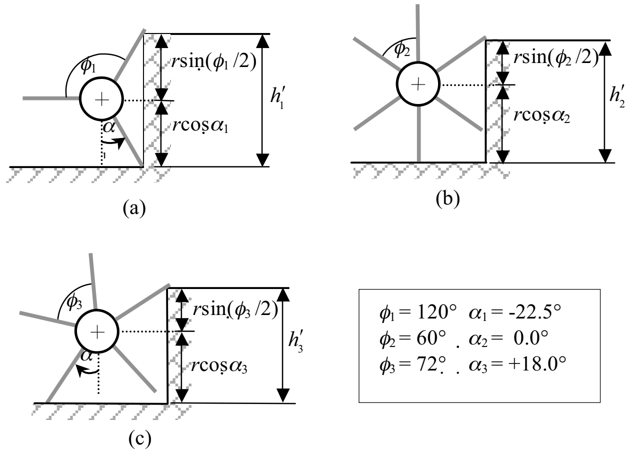

Fig. 3 Initial conditions for spoke wheel with
(a) Three (b) six and (c) five strips

Note that the initial condition plays an important role in climbing ability for spoke wheel model cases. Having different number of spokes produces different initial positions to obtain highest climbing ability. Fig. 3 demonstrates the initial conditions yielding the maximum heights for climbing over obstacles for spoke wheels with different number of strips. Three, six and five strips are chosen to represent the angle α for positive, neutral and negative directions.

2. *Static Friction Condition*

For a system in which the input torque and the coefficient of friction are specified, they may not satisfy the conditions for maximum geometric climbing ability. For these circumstances, quasi-static analysis is utilized to arrive at the maximum obstacle heights. Two models are introduced in static friction condition analysis, i.e., rigid-body and compliant spoke models.

2.1 Rigid body spoke model

Fig. 4 shows the force balance conditions at the state where the wheel starts to climb over an obstacle for normal and spoke wheels. In this figure, F_1 is the horizontal component of force the wheel received from the body, N_1 is the load from the body weight, F_2 is the frictional force, N_2 is the normal force occurring between the wheel and the step, T is an applied couple from the motor, and μ is the static coefficient of friction between the wheel and the step obstacle.

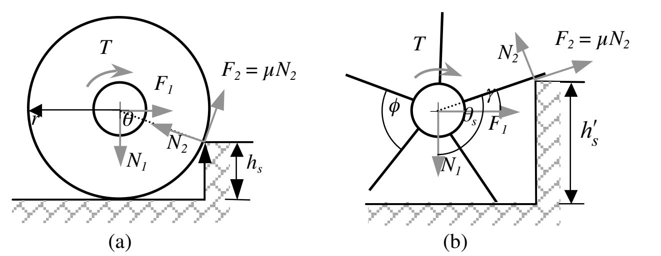

Fig. 4 Force balance conditions at the climbing state for (a) normal wheel (b) spoke wheel

Takahashi et al. [9] derived the maximum angle θ from force balance conditions and a static friction condition in which the wheel starts to climb up as

$$\theta = \cot^{-1}\left(\frac{1-\mu\varepsilon}{\mu+\varepsilon}\right) \tag{2}$$

Where $0 < \theta \leq 90^{\circ}$ and ε is the ratio of F_1 and N_1

If one knows μ and ε, the maximum step height h_s for wheel mechanism can be determined. Hence, the wheel can start to climb up the step taller than r ($\theta > 90°$) when $\mu\varepsilon > 1$.

From Fig. 4 (b), let γ be the angle between the horizontal line and the tangential line of the spoke tip touching the obstacle's top surface. The angle γ, that establishes the maximum obstacle height for spoke wheel h'_s, can be calculated in a similar manner as

$$\gamma = \cot^{-1}\left(\frac{1-\mu\varepsilon}{\mu+\varepsilon}\right) \quad (3)$$

Note that for the spoke wheel case, the maximum climbing angle $\theta_s = \gamma + 90^o$ where $0 \le \gamma \le \phi/2$. As a result, the angle θ_s is generally greater than 90 degrees and climbing ability of spoke wheels is automatically higher than their wheel radii. The maximum step height for spoke wheel h_s' is calculated by

$$h_s' = r(\sin\gamma + \cos\alpha) \quad (4)$$

It can be seen from (3) and (4) that different coefficients of friction and input thrust forces introduce different initial conditions to obtain the maximum permitted height. The maximum climbing ability is achieved when γ is equal to ϕ /2. This condition can be used to compute the required coefficient of friction when the applied force is known and vice versa.

For a particular instance where there is no applied force ($\varepsilon = 0$), (3) deduces to $\mu_0 = \tan\gamma$, which constitutes the static coefficient of friction for a sloped floor of angle γ.

2.2 Compliant spoke model

Each spoke is analyzed as a cantilever beam of length r subject to a point load P (perpendicular to the beam axis) at the free end. The maximum deflection δ at the tip can be computed as $\delta = Pr^3/3EI$ where E is modulus of elasticity of the beam and I is the moment of inertia of the beam section.

Five-spoke wheel is chosen to demonstrate the effect of compliant strip in reducing the height of the hub as shown in Fig. 5. In this case, there are two contact points, O_1 and O_2, to consider the compliant effect. We then take moment about O_1 and O_2 to solve for the normal forces N_2 and N_3 as shown in (5) and (6), respectively.

$$N_2 = \frac{T - N_1 r\sin\alpha}{r - \mu r} \quad (5)$$

$$N_3 = \frac{-T + N_1 r\cos\gamma}{r(\cos\gamma - \sin\alpha) - \mu h_s'} \quad (6)$$

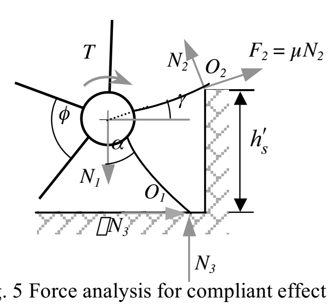

Fig. 5 Force analysis for compliant effect

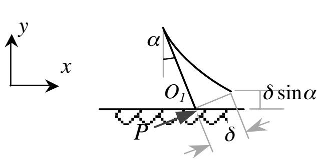

Fig. 6 Vertical deflection at point O_1

The point load at the point O_1 is calculated as $\mu N_3\cos\alpha + N_3\sin\alpha$. The deflection in P direction can be obtained. Then one can calculate the deflection in y direction as $\delta\sin\alpha$ as shown in Fig. 6. The hub height is lowered by this amount.

There are also the case of more than one contact points on the ground. This makes the calculation to be more complicated and it is not shown here.

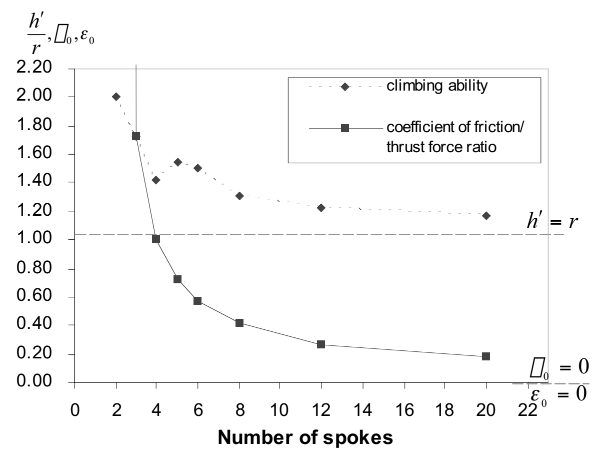

Fig. 7 Relationships between h'/r, μ_0 and ε_0 versus the number of spokes

The graph in Fig. 7 shows relationships between the ratio of geometric maximum height and the spoke wheel radius (h'/r), the required coefficient of friction to achieve h' without applied force (μ_0), the required thrust force ratio to achieve h' when $\mu = 0$ (ε_0), and the number of spokes. Since the values of μ and ε in (3) are interchangeable, they yield analogous curves. Theoretically, a spoke wheel with 2 strips can travel over the highest obstacle height of $2r$ with either required friction coefficient or applied force approaching infinity. However, the instance is unstable and is not possible in practice. From Fig. 5, it seems that 3 to 6 spokes should give more absolute heights than other cases. When the number of spokes is more than six, climbing ability decreases and the maximum height of climbing is approaching the spoke-wheel radius r as the number of spokes increases. The more number of spokes, the less required coefficient of friction.

III. Vehicle Design

For simplicity, this vehicle is designed to have spoke-wheel where the number of spokes can be adjusted. The enclosed body allows it to run in outdoor environment. The vehicle body is 24 cm long, 35 cm wide and 10 cm high. The radius r measured from hub center to spoke tip is 12 cm. Total body weight including 12 V battery is about 7.5 kg. It uses two gear head DC motors 7.2 V model 380K75 made by TAMIYA. The vehicle is controlled by wired joystick. Each motor is driving a pair of spoke wheels on each side via a sprocket chain mechanism with a teeth ratio of 9: 36. The vehicle is also designed to be able to move in an upward position. The battery and motor control are on-board with a wired remote control.

Components of the compliant spoke wheel are depicted in Fig. 8. The wheel consists of an inner hub, an outer hub, and removable spoke strips. The hub is designed such that the steel strips up to twelve strips can be inserted evenly along the hub circumference. The strips are made of 0.7 mm thick cold-rolled steel plates DIN1625. The spokes are also coated with synthetic rubber to increase friction when climbing.

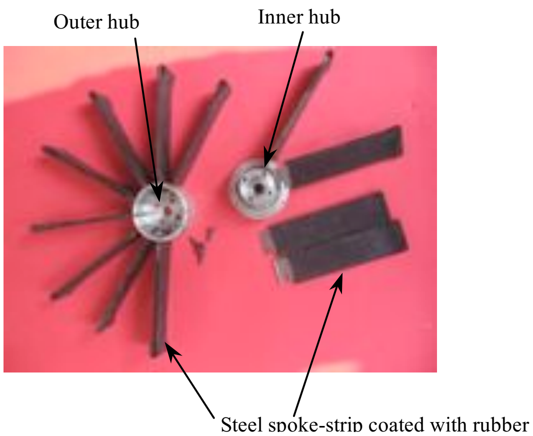

Fig. 8 Components of the spoke wheel

IV. Experimental Results

The experiment in searching the maximum permitted height of 4, 6 and 12 spokes are performed to compare with the calculation along with (4). The vehicle was tested on climbing concrete stairs by varying the height of support. Coefficient of friction between the strip and obstacle surface used in the calculation is 0.2 and the ratio ε is assumed to be 0.1.

Fig. 9 Four-spoke wheel climbing up the stairs

Four-spoke vehicle shown in Fig. 9 demonstrates the ability to climb over an obstacle 12 cm high which is less than the expected value of 15.2 cm. This can be the result of the deflection of the strips due to the vehicle weight. It can be observed that the body height of the vehicle measured from the hub center relative to ground is less than the wheel radius. In this case, the body height is 7 cm. The vehicle with six-spoke wheels can climb over 13 cm, which is higher than its hub as shown in Fig. 10. For the case of six-spoke wheel, body height of the vehicle is about 9.9 cm.

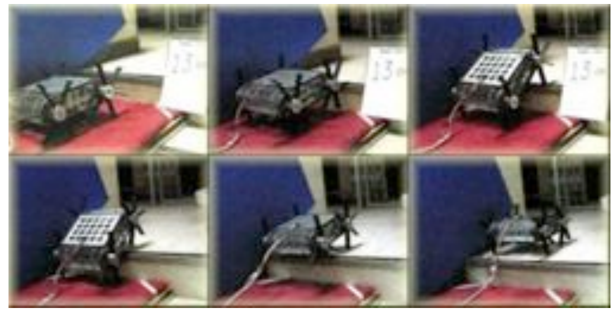

Fig. 10 Six-spoke wheel climbing up the stairs

Twelve spokes can climb an obstacle height of 13.5 cm shown in Fig. 11 whereas the body height is about 11.8 cm. Noting that experimental climbing ability is quite steady with the number of spokes equal to six.

Fig. 11 Twelve-spoke wheel climbing up the stairs

Comparison of climbing ability between the calculated values from (4) and experimental results are shown in Fig 12. The values from both methods show the same tendency in which climbing ability increases when number of spokes increases.

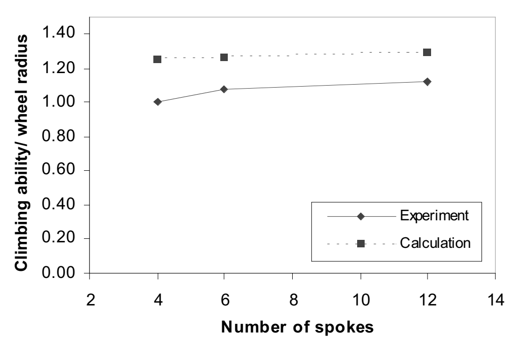

Fig. 12 Experimental results on the climbing ability of 4, 6 and 12 spokes compared to calculated values

For all the three cases studied, the values from experiments are less than the calculated values. The difference between the two values is the largest for four-spoke wheel that shows substantial deflections of spoke strips.

Better approximations of climbing ability for compliant spoke wheels can be achieved by considering the effects of the vehicle weight to the deformation of the spoke strips.

Fig. 13 shows the vehicle is climbing over a specially constructed disaster site of Robocup Rescue league in Thailand. In this case, the obstacle is made of wood. Six compliant spokes is chosen to test ride over the site and it can pass over the site without difficulty.

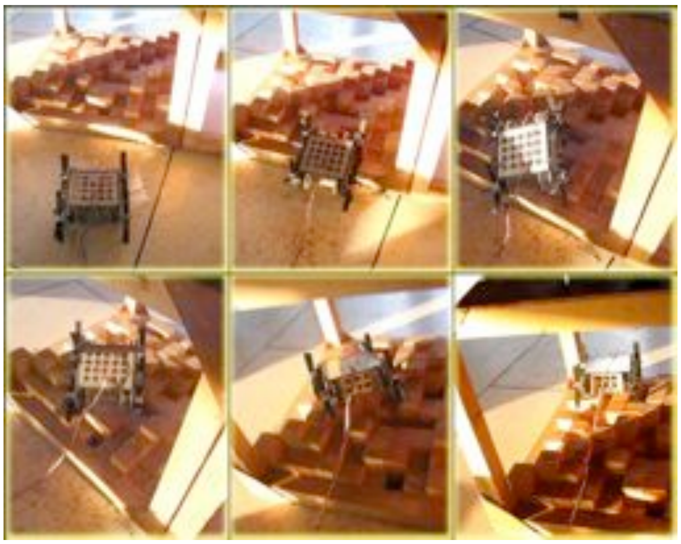

Fig. 13 Six-spoke wheel traversing across Robocup Rescue site

IV. Discussions

In this research, we proposed a method of climbing analysis for spoke wheel mechanism based on quasi-static principles. A spoke-wheel hybrid vehicle for enhancing stair climbing ability is successfully built to study the advantages of wheels with different number of spokes. For the constructed vehicle, the number of spokes can be adjusted by changing the number of strips attached to the wheel hub to obtain the optimum climbing ability.

Theoretically, less number of spokes promotes the ability of the vehicle to climb at a higher level of obstacle. However, the flexible spokes lower the body height which results in decreasing climbing ability. Less number of spokes also requires high coefficient of friction between the spoke tip and the obstacle surface. On the other hand, although the climbing ability over an obstacle is compromised, a larger number of spokes with compliant parts take advantages of having lower requirement for coefficient of friction. The availability of the proposed analysis is confirmed by experimental results.

The experimental results showed that all four, six and twelve spokes with compliance can climb over than their hub radius whereas twelve spokes give the maximum height of climbing among these three cases. Getting more spokes such as six and twelve can have more supported legs to distribute the same load. It also lifts the body high and is able to climb over higher level of obstacle. The results also showed that climbing ability of six spokes is slightly less than twelve spokes. Heuristically, six spokes consume less amount of power than twelve spokes, and then it should be more practical in traversing over various terrain. In addition, the construction of six spokes is simpler than twelve spokes.

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References

[1] Raibert, M.H., 1986. Legged Robots That Balance. The MIT Press, Cambridge, Massachusetts.

[2] Hirose, S., 1984. A study of Design and Control of a Quadruped Walking Vehicle. The International Journal of Robotics Research, 3 (2), pp. 113-133]

[3] Martin-Alvarez, A., de Peuter, W., Hillebrand, J., Putz, P., Matthyssen, A., and de Weerd, J., 1996. Walking robots for planetary exploration missions. Second World Automation Congress WAC96, Montpellier, France, May 27-30.

[4] Moore, E.Z., Campbell, D., Grimminger, F., and Buehler, M., 2002. Reliable stair climbing in the simple hexapod 'rhex'. IEEE Int. Conf. on Robotics and Automation (ICRA), Vol. 3, Washington, D.C. USA, pp. 2222-2227.

[5] Altendorfer, R., Moore, N., Komsuoglu, H., Buehler, M., H. B. Jr., McMordie, D., Saranli, U., Full, R., and Koditschek, D., 2001. Rhex: A biologically inspired hexapod runner. Autonomous Robots, Vol. 11, p.207.

[6] Quinn, R.D., Nelson, G.M., Bachmann, R.J., Kingsley, D.A., Offi, D.A. and Ritzmann, R.E., 2001. Insect designs for improved robot mobility. 4th Int. Conf. on Climbing and Walking Robots CLAWAR2001, Karlsruhe, Germany, pp. 69-76.

[7] Lauria, M., Estier, T., Siegwart, R., May 24-28, 2000. Innovative Space Rover with Extended Climbing Abilities. In Video Proceedings of the 2000 IEEE International Conference on Robotics and Automation, Sanfrancisco.

[8] Allen, T.J., Quinn, R.D., Bachmann, J., and Ritzmann, R.E., October 2003. Abstracted biological principles applied with reduced actuation improve mobility of legged vehicles. Proc. of 2003 IEEE/RSJ International Conference on Intelligent Robots and Systems, Las Vegas, Nevada, pp. 1370-1375.

[9] Takahashi, M., Yoneda, K. and Hirose, S., May 2006. Rough Terrain Locomotion of a Leg-Wheel Hybrid Quadruped Robot. Proc. Of the 2006 IEEE International Conference on Robotics and Automation, Oralando, Florida, pp. 1090-1095.